

The effect of deformation and vibration on the alpha decay half-life

O. N. Ghodsi, E. Gholami

*Department of Physics, Faculty of Science, University of Mazandaran
P. O. Box 47415-416, Babolsar, Iran*

Abstract

In this work, we expand upon our previous study, the effect of surface vibrations (low-lying vibrational states) on the calculation of penetration probability in α decay of spherical isotopes [Phys.Rev.C91,034611(2015)]. To this aim, the Coulomb and proximity potential model, taking in to account the ground state deformations of the involved nuclei along with the surface vibrations in the daughter nucleus, is used to evaluate the alpha decay probability. The results are compared with those obtained by spherical potential barrier, which shows the dramatic effect of employing the ground state deformations in case of deformed nuclei. As well, including of surface vibrations give rise to an increase in the value of tunneling probability in better agreement with experimental data.

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I. INTRODUCTION

Alpha decay is an important mode of radioactive decay to provide reliable information on the nuclear structure and to the identification of new elements[1-3]. So, experimental and theoretical calculation of alpha decay half-life has maintained particular interest in nuclear physics.

A simple theoretical way to determine the alpha decay width is by assuming the alpha particle to penetrate through a one dimensional potential barrier; and the penetration probability can be obtained in terms of the Wentzel-Kramers-Brillouin (WKB) semiclassical approximation [4-7]. The knowledge of this potential barrier which consists of long range coulomb interaction, short range nuclear interaction and centrifugal term is essential for the reasonable prediction of alpha decay half life. Different approximations give rise to a variety of models with different accuracy. So far different theoretical models such as shell model, the fission like model, and the cluster model have been used to determine the potential barrier governing the alpha decay process [8-15]. The role of some important factors like the shell effects have been studied by Tonozuka and Arima [16] and the fine structure of alpha decay has been reported by Santhosh et al.[17]. Moreover, Royer taking account of the role of angular momentum, proposed analytic formulas for alpha decay half life[18]. Deformation is also an important factor and various models incorporated the effects of deformation and orientation on the interaction potential [12,15,17,19]. In the present work, in order to consider the effect of deformation and orientation degrees of freedom, we perform calculations in the frame work of coulomb and proximity potential model for deformed nuclei [20].

In a recent study [21], we considered the vibrational low lying states of nuclei with spherical equilibrium shape in their ground states, and studied the effect of these deformations on penetration probability by making use of two different approaches- semiclassical and quantum mechanical. As well, It is well-known that in case of a deformed parent there are many accessible ground states and low lying excited states in the daughter nucleus; So in the next step we aim to consider the coupling of these states during tunneling process through the deformed potential barrier.

To this aim, at first, taking the coulomb and proximity potential model proposed by Shi and Swiatecki [22] as interacting barrier, the involved nuclei are considered as spheres; and the effect of vibrational low lying states of daughter nuclei on penetrability through this spherical barrier is calculated. Then, in the following, the ground state deformations (β_2 and β_4) of parent and daughter nuclei are also incorporated to improve the above mentioned interaction potential. This modified version of coulomb and proximity potential model for deformed nuclei (CPPMDN) then is used for the calculation of alpha decay penetration probability and to investigate the effect of low lying excited states. This provides an opportunity to see the effect of ground state deformations as well as the vibrational low lying states of daughter nuclei on the half life time calculations.

This paper is organized as follows: In section II the formalism of interaction potentials used and the formulas of calculating alpha decay half life are briefly described; and in the last part of this section, the effects of vibrational states on the decay process are considered. The results are given in section III.

II. THEORETICAL FORMALISM

The potential barrier governing the alpha emission is a key component in calculation of penetration probability. The present paper deals with two cases: (i) all the involved nuclei treated as spherical, (ii) the parent and daughter nuclei have an axially symmetric deformation. In the second case the potential barrier depend on the the polar angle between the symmetry axis of parent or daughter nuclei and the direction of alpha emission. The half life of alpha decay process is then calculated in the frame work of quantum mechanical tunneling using the WKB approximation.

The half-life time is calculated as :

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{P\nu}, \quad (1)$$

where λ , the decay constant, is simply the product of the barrier penetration probability (P) and the assault frequency (ν). The assault frequency, the number of assaults on the barrier per second is evaluated from empirical zero point vibration energy E_ν in the harmonic oscillator approximation. E_ν for alpha decay is proportional to Q value as : $E_\nu = 0.095Q$ [23]. Q is the released energy at alpha decay. The penetration probability of alpha particle is taken equal to one. We have used the method of evaluation of fusion probability for the calculation of tunneling probability in alpha decay.

P is evaluated by averaging the penetration probability over the polar angle between the symmetry axis of axially-symmetric deformed parent or daughter nuclei and the direction of alpha emission,

$$P = \int_0^{\pi/2} P_\theta \sin(\theta) d\theta. \quad (2)$$

P_θ , the penetration probability in the WKB approximation can be written as :

$$P_\theta = \frac{1}{1 + \exp\left[\frac{2}{\hbar} \int_{a(\theta)}^{b(\theta)} \sqrt{2\mu(V(L, \theta) - Q)} dr\right]}, \quad (3)$$

where $a(\theta)$ and $b(\theta)$ are the turning points obtained from the equation $V(L, \theta) = Q$. μ is the reduced mass, $\mu = m \frac{A_\alpha A_d}{A_\alpha + A_d}$, where m is the nucleon mass; A_α and A_d are the mass numbers of alpha particle and daughter nucleus respectively.

In Coulomb and proximity potential model for deformed nuclei the interaction potential $V(L, \theta)$ is considered as the sum of coulomb repulsion, nuclear proximity potential and centrifugal parts for the post-contact configuration. The potential barrier for the overlap region is then constructed by a smooth, power-law interpolation between the contact and parent configuration [20].

$$V(L, \theta) = V_c(r, \theta) + V_p(z) + \frac{l(l+1)\hbar^2}{2\mu r^2} \quad L > L_c \quad (4)$$

$$V(L, \theta) = a(L - L_0)^n \quad L_0 < L < L_c \quad (5)$$

L is the overall length of the configuration, L_c is the length of the contact configuration and L_0 is the diameter of the parent nuclei. r is the distance between fragment centers and z is

the distance between the near surfaces of the fragments. l represents the angular momentum and μ is the reduced mass. The constants a and n are determined by the requirement of smooth fit of two potentials at the touching point.

$V_p(z)$ is the proximity potential, given by

$$V_p(z) = 4\pi b\gamma \frac{C_1(\theta)C_2}{C_1(\theta) + C_2} \phi\left(\frac{z}{b}\right), \quad (6)$$

γ is the nuclear surface tension coefficient :

$$\gamma = 0.9517 \left[1 - 1.7826 \left[\frac{N - Z}{A}\right]^2\right]. \quad (7)$$

N , Z and A are the neutron, proton and mass numbers of parent nucleus. $C_1(\theta)$ and C_2 are the radius vectors of the daughter and alpha particle respectively.

$\phi(\frac{z}{b})$ the universal function is given as [24]:

$$\phi(\xi) \approx -4.41 \exp(-\xi/0.7176) \quad \xi \geq 1.9475 \quad (8)$$

$$\phi(\xi) \approx -1.7817 + 0.9270\xi + 0.0169\xi^2 - 0.05148\xi^3 \quad 0 \leq \xi \leq 1.9475 \quad (9)$$

$\xi = \frac{z}{b}$, where b is the diffuseness of the nuclear surface ($b \approx 1fm$). See Ref.[20] for more details.

The Coulomb interaction between the two deformed and oriented nuclei:

$$V_c(r, \theta) = \frac{Z_1 Z_2 e^2}{r} + 3Z_1 Z_2 e^2 \sum_{\lambda, i=1,2} \frac{1}{2\lambda + 1} \frac{(R_i(\alpha_i))^\lambda}{r^{\lambda+1}} Y_\lambda^0(\alpha_i) [\beta_{\lambda i} + \frac{4}{7} \beta_{\lambda i}^2 Y_\lambda^0(\alpha_i) \delta_{\lambda,2}] \quad (10)$$

where,

$$R_i(\alpha_i) = R_{0i} [1 + \sum_{\lambda} \beta_{\lambda i} Y_\lambda^0(\alpha_i)] \quad (11)$$

and

$$R_{0i} = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3}. \quad (12)$$

The ground state deformations of the involved nuclei (β_2 and β_4) are incorporated in the evaluation of the total potential. According to the above mentioned formalism, using WKB approximation the penetration probability has been calculated from Eq.(3).

In the following to address the effects of accessible collective modes and the dependence of penetration probability on the low-lying vibrational excitations, the coupled channel formalism is employed by including all the relevant channels and assuming the harmonic oscillator for vibrational coupling. The stationary coupled Schrodinger equation can be written as [25]:

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N^0(r) + \frac{Z_P Z_T e^2}{r} + \varepsilon_n - E\right] \psi_n(r) + \sum_m V_{nm}(r) \psi_m(r) = 0, \quad (13)$$

where V_{nm} are the elements of coupling Hamiltonian [25], and V_N^0 is the potential between two interacting nuclei. Here a Woods-Saxon potential which fitted to the coulomb and proximity potential model in each configuration is used for the interaction potential. The excitation energy appears as ε_n . The above coupled channel equations are solved under the condition that there are only incoming waves at $r = r_{min}$, the starting point of integration, which is taken as the minimum position of the coulomb pocket and there are only outgoing waves at infinity for all channels except the entrance channel:

$$\Psi_n(r) \rightarrow T_n \exp(-i \int_{r_m}^r k_n(r') dr') \quad r \rightarrow r_{min} \quad (14)$$

$$H_J^-(K_n r) \delta_{n0} + R_n H_J^+(K_n r) \quad r \rightarrow r_\infty \quad (15)$$

practically the numerical solution is matched to a linear combination of incoming and outgoing wave function where both the nuclear potential and coulomb coupling are sufficiently small. Reflection and transmission coefficients in each channel are denoted by R_n and T_n , respectively. H_J^- and H_J^+ are the incoming and outgoing coulomb functions. at $r = r_{max}$, by superposing of the incoming and outgoing coulomb waves:

$$\chi_{nm}(r) = C_{nm} H_J^-(K_m r) + D_{nm} H_J^+(K_m r) \quad r \rightarrow r_{max} \quad (16)$$

and the solution of the coupled channel equations with the proper boundary condition Eq.(14) and Eq.(15) are:

$$\Psi_m(r) = \sum_n T_n \chi_{nm}(r), \quad (17)$$

and at r_{max} ,

$$\Psi_m(r_{max}) = \sum_n T_n \chi_{nm}(r_{max}), \quad (18)$$

by comparing with Eq.(15):

$$\sum_n T_n C_{nm} = \delta_{m0}, \quad (19)$$

then, the penetrability correspond to every angle is given by:

$$P_\theta = \sum_n \frac{K_n(r_{min})}{K_0} |T_n|^2. \quad (20)$$

See Ref.[25] for more details.

In this study in order to identify the role of coupling effects on the calculation of penetration probability, the CCFULL code is used, considering one phonon excitations of 2^+ states of daughter nuclei. Since the α particle is a closed-shell nucleus and has a high lying excited state of 20 MeV, its excitations are not included. deformation parameters and excitation energies at 2^+ excitation state of daughter nuclei used in the input file of CCFULL code are given in Table 1.

III. RESULTS

As pointed out in the Introduction, in a previous study the role of coupled channel effects demonstrated in case of nuclei with spherical equilibrium shape in their ground state [21]. In the present investigation the Coulomb and proximity potential model is applied to the calculation of alpha decay half-life of ^{162}Hf , ^{174}Os , ^{226}Ra , ^{226}Th , ^{226}U , $^{176,178,190}Pt$

and $^{188,190}Pb$ isotopes. The alpha emitters are taken such that the deformation and intrinsic properties such as vibrations and corresponding deformations to be considered. As a first step, all the involved nuclei are considered spherical and penetrability through a spherical barrier is calculated based on semiclassical WKB approximation. In the second step considering the crucial role of ground state deformations(β_2 and β_4) of parent and daughter nuclei, we extend this spherical potential barrier to the Coulomb and proximity potential model for deformed nuclei(CPPMDN); The potential barrier so constructed incorporates ground state deformations, and half-life time value is found to decrease in comparison to the spherical case (see columns two and four of table II). In order to see the influence of vibrational excitations on penetration probability, the coupled channel approach is used considering the effects of coupling to the low-lying 2^+ excitation state of daughter nuclei in both the above mentioned cases i.e spherical and deformed potential. The deformation parameters of these nuclei used in CCFULL code taken from Ref.[26], are given in Table I. In table II the logarithm of half-life time calculated from one dimensional potential and coupled channel approach for both the spherical and deformed potentials are presented. It is evident that including of surface vibrations has dramatic influence on the half-life value. In fact these couplings give rise to an increase in the value of tunneling probability due to the reduction of potential barrier height. Alpha decay half-lives of the above mentioned nuclei have been calculated in many recent studies. Denisov and Khudenko [27]evaluated alpha decay half-lives of 344 nuclei with the help of empirical relations in the frame work of unified model for alpha decay and alpha capture (UMADAC) ; Xu and Ren [28] presented a systematic calculation using microscopic density dependent cluster model(DDCM); And by using generalized liquid drop model (GLDM) Bao et al.[29] investigated the role of shell effects in the behaviour of decay half-lives. Corresponding values of half-lives from Refs.[27-29] along with the experimental values are presented in the last four columns of Table 2. From the table it may be seen that by considering the ground state deformations (CPPMDN) the results get modified in comparison to spherical potential (CPPM) and by taking into account the ground state deformations along with surface vibrations(CPPMDN + CC) shows better agreement with experimental data.

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Table 1. Deformation parameters and excitation energies at $J^\pi = 2^+$ excitation state of daughter nuclei used in the Coupled Channel calculations [26].

Nuclei	$E_{ex.}(MeV)$	β_{vib}
^{158}Yb	0.3582	0.194
^{170}W	0.1568	0.24
^{222}Rn	0.1862	0.141
^{222}Ra	0.111	0.192
^{222}Th	0.183	0.153
^{172}Os	0.227	0.225
^{174}Os	0.158	0.226
^{186}Os	0.137	0.2
^{184}Hg	0.366	0.16
^{186}Hg	0.405	0.132

Table 2. The decimal logarithm of alpha decay half life through one dimensional potential (ODP) and coupled channel approach (CC) using two versions of coulomb and proximity potential model: spherical (CPPM) and deformed (CPPMDN). Experimental values along with corresponding results predicted by the unified model for alpha decay and alpha capture (UMADAC)[27], density-dependent cluster model(DDCM)[28], and generalized liquid drop model (GLDM)[29] are also presented.

	CPPM		CPPMDN		UMADAC	DDCM	GLDM	Exp.
	ODP	CC(2 ⁺)	ODP	CC(2 ⁺)				
^{162}Hf	6.878	6.579	6.599	6.2	5.86	—	6.17	5.8
^{174}Os	6.233	5.796	5.803	5.224	5.27	5.38	5.447	5.34
^{226}Ra	12.393	12.01	11.697	11.213	11.28	—	11.164	10.73
^{226}Th	4.566	4.201	4.02	3.504	3.58	—	3.44	3.39
^{226}U	0.395	0.141	0.022	-0.377	-0.18	—	-0.49	-0.57
^{176}Pt	2.040	1.658	1.746	1.239	1.26	1.415	1.415	1.22
^{178}Pt	3.477	2.992	3.054	2.410	2.56	2.813	2.76	2.45
^{190}Pt	20.381	19.9747	19.935	19.4359	19.22	—	19.255	19.31
^{188}Pb	2.829	2.585	2.710	2.407	2.22	—	2.04	2.06
^{190}Pb	4.7581	4.535	4.637	4.401	4.13	—	3.88	4.25